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Numerical Study of Material Carrier Car on a Belt Conveyor Using the Totally-Asymmetric Simple Exclusion Process with Parallel Updating and Periodic Boundary Condition

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Abstract—An interesting process in material industries is the distribution process of the materials. The distribution of materials in large numbers is done using a material carrier car into silos. When the distribution of materials into one silo is full, the distribution to the next silo may be delayed. This may cause a loss to the company due to ineffectiveness and inefficiency of the distribution process. This article discusses a mathematical modeling in the distribution of materials using a carrier car towards the silos on a belt conveyor. The mathematical model utilized is the totally-asymmetric simple exclusion process (TASEP). TASEP in one dimension (1D) is a stochastic process in which hard core particles that occupy 1D lattice jump to their nearest neighbor as long as the nearest neighbor is not occupied by another particle. Using this model, the density and current density profiles of the TASEP are obtained that can describe the density and current density of the material carrier car heading to the silos. It is obtained from the TASEP model that the rate of the particle jumping is quite low.

Keywords: belt conveyor, material carrier car, parallel updating, periodic boundary condition, TASEP

I. INTRODUCTION

The development of industrial materials in Indonesia is followed by the demand and availability of the materials. In order to meet market demands, material industries require equipment to transport materials from the mining location to the processing plant, which is calls conveyor belt. A belt conveyor has a function to move a cargo in the form of units or bulks along a straight line or a limited angle of inclination. The movement of a belt conveyor is in the horizontal direction or forming an angle from one operating system to another operating system in one line of production processes. In addition, the belt conveyor is a machine that has a large enough capacity that is 500 up to 5000 m^3/hour or more. The conveyor belt is also able to transfer material to a relatively far distance between 500 to 1000 meter or more.

P.T. Well Harvest Wining Alumina Refinery Ketapang is a company whose productivities is in the field of purification of bauxite mining material into alumina. This company uses a belt conveyor to transport bauxite material from the mining location to the silo in the processing plant. A silo, also called a stockpile, is a place to store the bulk material. In this case, a silo is a place to store bauxite reserves in the bauxite temporary storage warehouse. To arrive at a silo, the bauxite materials are carried by a carrier car, by which the bauxite materials are poured into the silo. But the distribution of bauxite material using a conveyor belt into the silos often encounter obstacles in the form of equipment termination. Termination occurs due to the system hardware equipment which is not functioning properly. In addition, when the distribution of material into one silo is full, the conveyor belt will stop operating. These problems lead to delays in the distribution of material to the next silo.

To overcome these problems, a mathematical modelling of material distribution via a carrier car on a belt conveyor into the silo is used, namely the totally-asymmetric simple exclusion process or known as TASEP. TASEP in one-dimensional (1D) is a stochastic process in which hard core particles that occupy a 1D lattice sites move towards the nearest neighbor site as long as the neighbor site is not occupied by other particles [1,2]. The jump can occur in one direction only, i.e.: to the right or to the left. TASEP is a model of particle jumping which is used to study non-equilibrium systems. There are many scenarios given in the dynamics of TASEP such that the flow in the system is maintained. TASEP has been used to study various physics and biological phenomena, including protein synthesis [3], kinematics polymerization of macromolecules [4-6], and traffic of insects [7,8]. In addition, a wide variety of boundary conditions can be specified on the TASEP. One of them is the periodic boundary condition. The periodic boundary condition produces a ring-shaped geometry so that the number of particles in the lattice sites is constant. Thus, an equilibrium condition is obtained in the system by which particles randomly hop along the sites with the same probability. However, if the rate of the particle jumping is made inhomogeneous, a phase transition from low density to high density may occur.

In this article, the material carrier car is modeled as the hard core particle and the belt conveyor is modeled as the lattice site. Furthermore, it is assumed that the bauxite material is distributed from one silo to the next one sequentially, and experiencing some delay in the movement, so that the hard core particle has a low hopping rate to the lattice which consists of a silo. From this connection the density and current density of the particle may describe the density and current density of the material carrier car moving from one silo to another. To complete the TASEP model, a periodic boundary condition and dynamics of parallel updating are used.

The density and current density that are obtained by numerical methods using the C++ language program are based on the mean field approach (mean-field approximation). Finally, through the density and current density of the hard-core partcle, the physical behavior of the material carrier car carrying bauxite material around the silo can be studied. Hence, the objectives of this article are i) modeling the movement of material carrier car distributing bauxite materials on a belt conveyor using the TASEP model with periodic boundary condition and parallel updating dynamics, ii) determining the density and current density of one hard-core particle (material carrier car) on a lattice site (belt conveyor) with variation of distances between sites that have low hopping rates (indicating a silo) generated by the TASEP model, and iii) interpreting the density and current density of the hard-core particle as the density and current density of the material carrier car on the conveyor belt move bauxite materials into the silos.

II. METHOD OF RESEARCH

This is a theoretical research via a numerical methods. The model being studied is the TASEP. The TASEP is a mathematical model that can be used to study the dynamic behavior of a physical system. The physical systems mentioned here are systems that have moving elements (motile) and tracks in which the motile elements are moving upon. Here the TASEP is used to model the distribution system of a material carrier car on a belt conveyor which is moving bauxite materials into silos. If the material carrier car arrives in a silo, then the bauxite materials in the carrier car are transported and temporarily stored in this silo. For modeling the above distribution process, it is first necessary to build a relationship between the TASEP model and the system being studied. The linkage between the TASEP model with the system being studied can be identified as follows:

 i) the carrier car is represented as a single hard core particle,
 ii) the belt conveyor is represented as a lattice system, and
 iii) the silo is represented as the lattice site that has a low hopping rate.

From the three relations above, one may obtain a TASEP model where there is only one particle moving on the lattice system. In accordance with the periodic boundary condition, the particle jumps from one site to the next nearest neighbor site with a specific hopping rate. In addition, the setting of parallel updating dynamics causes the particle to jump from one site to the right nearest neighbor site with certainty, except at a certain site that consists of a silo. Because a material carrier car is quite heavy as it carries bauxite materials, the speed at which the carrier car is moving over the conveyor belt can also be realized as not too fast. It also keeps the belt conveyor not easily broken. Thus, the hopping rate when the carrier car is on the silo will experience a significant decline. Therefore, in this study the hopping rate when the particle has reached the silo is $k = 0.001$. This value is still quite small when compared to the total number of iteration steps $t = 10^6$ time steps. This means that the steady state can still be met. In general, this study obtains a TASEP model where there is only a single particle hopping in the lattice

system with periodic boundary condition. In addition, the hopping rate of the particle in this model is quite small, especially in the lattice containing a silo. This is what makes the TASEP modeling in this study quite interesting and different from other TASEP models.

Following the relationship between the distribution system and the TASEP above, a continuity equation is used via the TASEP model. These equations are solved numerically using the C++ language program, such that the steady state density and current density of the TASEP are obtained. Finally, from the density and current density obtained, interpretations for the distribution system are made based on the relationship aforementioned.

III.   RESULTS AND DISCUSSION

In this study, one interesting thing to learn is how the distance between the lattice containing the silos influences the density and current density of the system. The distance between the lattice (silo) will be varied for two conditions, namely five silos directly neighboring (adjacent to each other) and five silos separated by a distance of five usual lattice sites between two adjacent silos. The density and current density results as a function of lattice sites are obtained from a numerical method which can be shown in succession in Fig. 1 and 2.
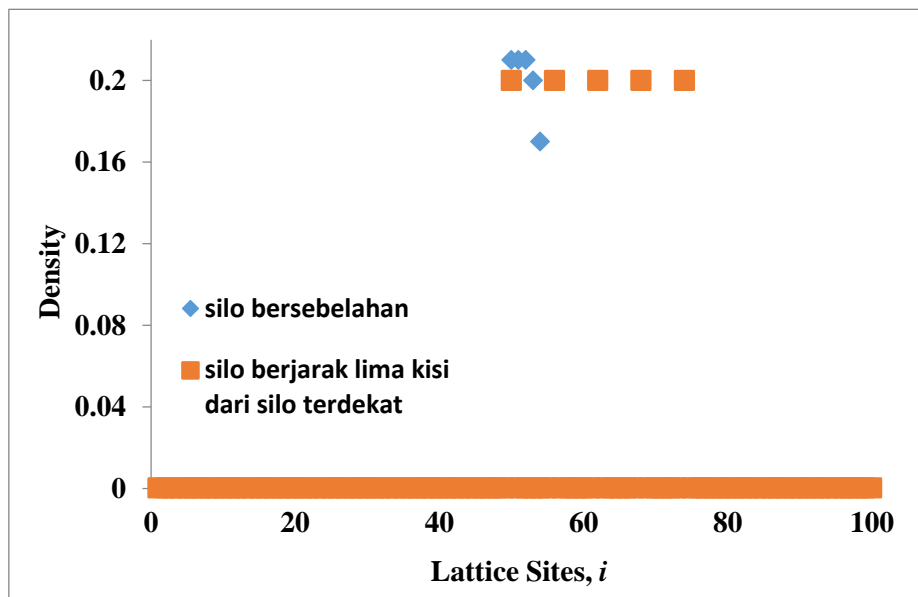

FIGURE 1. The density profile of the TASEP with parallel updating and periodic boundary condition which consists of one particle. The diamond (blue) data indicate the density profile where the lattice sites in which the hopping rate is reduced adjacent to each other. The square (orange) data indicate the density profile where the lattice sites in which the hopping rate is reduced distanced five lattice sites apart.

Fig. 1 shows the result of the numerical method for the density of a particle in the lattice system with periodic boundary conditions and variations in distance between the silos. The diamond (blue) data show the density profile for silos adjacent to one another. While the square (orange) data show the density of a particle for a silo within five lattice sites from another adjacent silo. The first silo is located in site $i = 50$. Generally, it can be observed that the density of the particle is very small (close to zero), either for silo adjacent or distant, except in the middle of the lattice sites. In the lattice containing a silo, the density rises to around 0.21 for the adjacent silos and 0.2 for distant silos. This shows that the distribution of the density is centered around the lattice sites that consist of silos. This makes sense, since on other lattice sites (not consisting of silos) the particle continue jumping with a hopping rate of 1.0 resulting in a low density. On the other hand, the low hopping rate on the lattice sites consisting of silos causes the density profile to be higher. Thus, the density profile in Fig. 1 is in a low density phase.

The highest density difference is fairly small between adjacent silos ($\rho = 0.21$) and distant silos ($\rho = 0.2$). This is also quite interesting. Although the difference is quite small, which is 0.01, it shows the difference in the density if the silos are adjacent to each other and when the silos are apart from one

another. The difference in the density indicates that the system tends to have higher density when the silos are installed adjacent to each other than when these silos are distant to each other. In other words, the particle tends to stay longer on the silos which are adjacent to each other than when these silos are spaced. In addition, it can also be observed that the density for the adjacent silos appears more regular than the density for the distant silos. Finally, around the area of the highest density for the adjacent silos, the density plummet around the last silo.
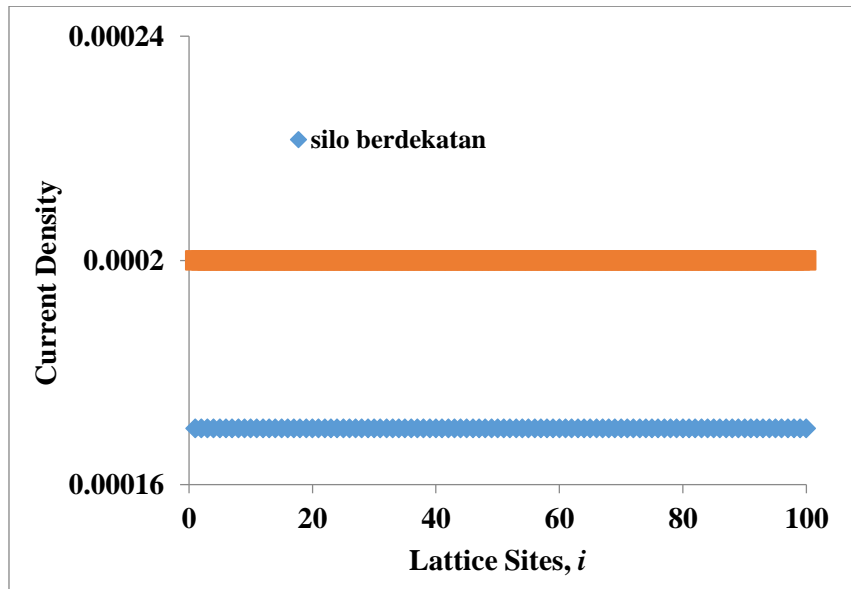

FIGURE 2. The current density profile of the TASEP with parallel updating and periodic boundary condition which consists of one particle. The diamond (blue) data are the density profile where the lattice sites in which the hopping rate is reduced are adjacent to each other. The square (orange) data are the density profile where the lattice sites in which the hopping rate is reduced are distanced five lattice sites apart.

Fig. 2 shows the current density of a particle moving in a lattice system with periodic boundary conditions and variations in the distance between the silos. The diamond (blue) data show the current density along the lattice sites for the adjacent silos. The square (orange) data show the current density along the lattice sites for distant silos. Interesting to note that the current density of the particle for the distant silos, $J = 0.0002$, is greater than the current density of the particle for silos adjacent to each other, $J = 0.00017$. This means that composing the silos distant away from each other makes the particle moving along the lattice sites become more fluent. This may be caused by the barrier faced by the particle with silos arranged apart from each other is smaller than if the silos are arranged close together. Here, the variations in the distance between the silos are made only in two variations. For longer separation distance between silos, the result of the density profile appears unchanged for the maximum density; only the distance between the density changes according to the composition of the silos.

Further discussion is given concerning the interpretation of the TASEP model. As a reminder, the system under consideration here is the distribution system of a material carrier car moving bauxite materials on a belt conveyor into silos. Following the identification above, the material carrier car transporting materials is the hard-core particle and the belt conveyor is the lattice system. Many interesting observations may be gained concerning the behavior of the material car carrier transporting materials that moves on the belt conveyor. First, the distance between the silos affects the density and current density of the material carrier car moving on the conveyor belt. Assuming the material carrier car transporting materials does not move too quickly on the belt conveyor, the greater the distance between the silos, the more fluent the material carrier car moves above the belt conveyor. Placement of silos that is too close to one another can lead to a longer time of the material carrier car to be in the area around the silos, hence making it less fluent in moving through the belt conveyor. Of course this could lead to a certain economic impact on the operating and maintenance costs of the belt conveyor system. In this case,

it is recommended that, in the arrangement of the distribution system, in silos temporary storage warehouse should be placed not too close to each other.

IV. CONCLUSION

A numerical study of a material carrier car carrying bauxite materials moving on a conveyor belt into silos has been presented. The model used to study the aforementioned distribution system is the TASEP with periodic boundary conditions and parallel updating dynamics. The TASEP consists of only one particle. The result shows that varying the distance between the silos affects the density and current density of the material carrier car moving on the belt conveyor. Placing the silos further apart from one another makes the current of the carrier car more fluent than placing the silos adjacent to each other.

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